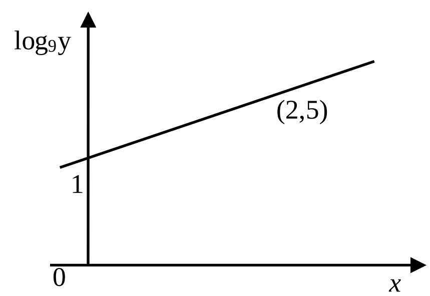
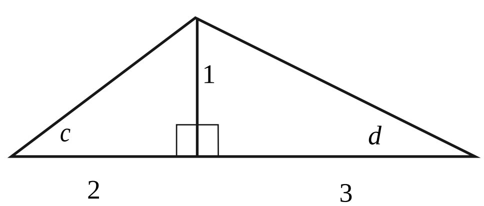
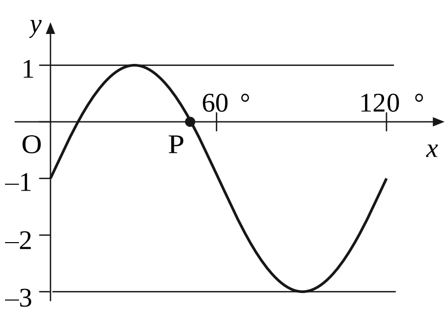
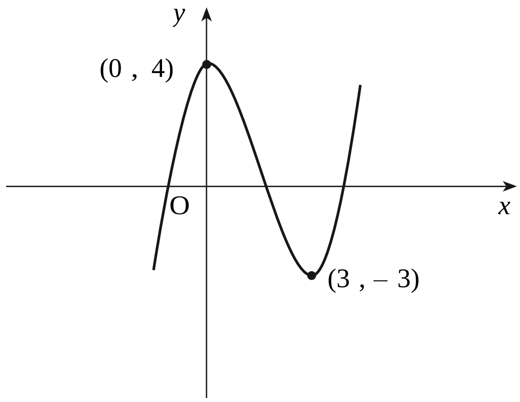
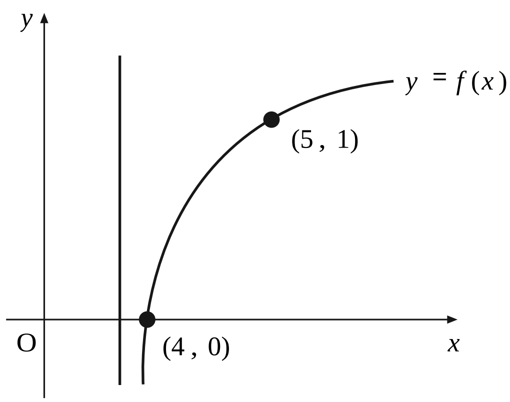
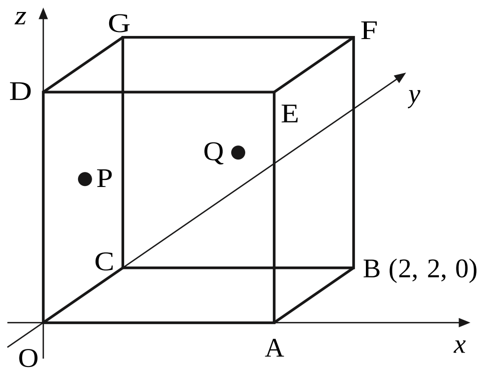
	Higher Revision – Unit 1 A/B Test	
1.	Solve the equation $\log_4(5+x)-\log_4(x)=2,\ \ x<3$	**4**
2.	Find the coordinates of the point where the curve with equation $y=\log_3(x-1)$ meets the straight line $y = 2$	**3**
3.	A cup of tea cools according to the law $T_t = T_0e^{-kt}$, where T_0 is the initial temperature and T_t is the temperature after t minutes. All temperatures are in °C (a) A particular mug of tea cools from boiling point (100°C) to 75°C in a quarter of an hour. Calculate the value of k (b) By how many degrees will the temperature of this tea fall in the next quarter of an hour	(a) **3** (b) **2**
4.	Evaluate $2-\log_5\left(\frac{1}{25}\right)$	**2**
5.	Two variables x and y are connected by the law $y = ba^x$. The graph of $\log_9 y$ against x is a straight line with a y-intercept of 1 passing through the point (2, 5) as shown in the diagram.  Find the values of b and a	**4**
6.	The diagram shows two right-angled triangles with angle c and d marked as shown  (a) Find the exact value of sin (c + d) (b) (i) Find the exact value of sin 2c (ii) Show that cos 2d has the same exact value	(a) **4** (b) **4**

7.	A curve has the equation $y = 3\sin x + \cos x$ (a) Express $y = 3\sin x + \cos x$ in the form $y = k\sin(x - a)°$ where $k > 0$ and $0 \le x \le 360^\circ$ (b) State the maximum value of the curve $y = \sin x + \cos x$, and the value of x at which this maximum occurs.	**4** **2**
8.	(a) Find an equivalent expression for $\cos\left(x - \frac{\pi}{6}\right)$ (b) Hence, or otherwise, determine the exact value of $\cos\frac{\pi}{12}$	**1** **3**
9.	A function f is given by $f(x) = \sqrt{9-2x}$ (a) State a suitable domain for this function (b) Find $f^{-1}(x)$, the inverse function to $f(x)$	**1** **2**
10.	Two functions are given as $f(x) = x^2 - 1$ and $g(x) = 2 - x$ (a) Find the composite function $f(g(x))$, give you answer in the form $ax^2 + bx + c$, (b) Solve $f(g(x)) = 8$	**3** **2**
11.	The diagram below shows part of the graph of a function whose equation is of the form $y = a\sin(bx°) + c$  Write down the values of a, b and c	**3**

12.	 The diagram shows part of the function $y = f(x)$ which has turning points at (0,4) and (3,-3) Sketch the graph of $y = -f(x-2)$	**3**
13.	The diagram below shows part of the log function $y = \log_a (x - b)$  Identify values for a and b	**2**
14.	(a) Express $3x^2 + 12x + 13$ in the form $a(x+b)^2 + c$ (b) Hence state the range of the function $y = 3x^2 + 12x + 13$	**2** **1**
15.	Given that vector **u** = -3***i*** +4***k*** (a) Find the magnitude of vector ***u*** (b) State the components of a unit vector parallel to ***u***	 **1** **1**
16.	The point P divides AN in the ratio 4: 1. Given that A is (-5, 6, -5) and N is (10, -4, 0), find the coordinates of P	**3**

17.



OABCDEFG is a cube with side 2 units, as show in the diagram

B has coordinates (2,2,0)

P is the centre of the face OCGD and Q is the centre of face CBFG

(a) Write down the coordinated of G **1**

(b) Find ***p*** and ***q*** the position vectors of points P and Q **2**

(c) Find the size of the angle POQ **5**

18. Given that vectors ***a***, ***b*** and ***c*** are defined as follows

$$\boldsymbol{a} = \begin{pmatrix} 2 \\ 0 \\ -1 \end{pmatrix}, \quad \boldsymbol{b} = \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix} \text{ and } \quad \boldsymbol{c} = \begin{pmatrix} 0 \\ -1 \\ 1 \end{pmatrix}$$

(a) Evaluate $\boldsymbol{a} \bullet \boldsymbol{b} + \boldsymbol{a} \bullet \boldsymbol{c}$ **2**

(b) From your answer to part (a) make a deduction about the vector $\boldsymbol{b} + \boldsymbol{c}$ **3**

answers	
1.	$\log_4\left(\frac{(5+x)}{(x)}\right) = 2, \quad \frac{(5+x)}{(x)} = 4^2, \quad 5+x=16x, \quad x=\frac{1}{3}$ Example 1.18 on page 13, Question 1 on page 13
2.	$2=\log_3(x-1), \quad 3^2 = x-1, \quad x=10 \quad (10,2)$ Example 1.13 on page 11 Question 2 on page 12
3.	(a) $75 = 100e^{-kt} \rightarrow \ln(0.75) = -kt \rightarrow k = 0.0192$ (b) $T_{15} = 75e^{-0.0192\times 15}$ *or* $T_{30} = 100e^{-0.0192\times 30}$, temp falls by 18.75°C Example 1.23 on page 16 Questions 3 – 5 on page 17
4.	$2-\log_5\left(\frac{1}{25}\right) = 2-\log_5\left(5^{-2}\right) = 2-{}^{-2}\log_5 5 = 4$ Example 1.8 and 1.9 on page 9 Question 3 on page 9
5.	Take $\log_9$ for both sides of $y = ba^x$ $\log_9 y = \log_9 ba^x$ $\log_9 y = \log_9 b + \log_9 a^x$ $\log_9 y = \log_9 b + x\log_9 a$ $\log_9 y = x\log_9 a + \log_9 b$ Find the equation of the straight line $\log_9 y = 2x + 1$ Compare with $\log_9 y = x\log_9 a + \log_9 b$ $x\log_9 a = 2x$, $\log_9 a = 2$, $a = 9^{2}$, $a = 81$ $\log_9 b = 1$, $b = 9^1$, $b = 9^1$ Therefore the original exponential function is $y = 9\times 81^x$ Example 1.26 on page 20 Question 2 on page 21

6.	$\sin c = \frac{1}{\sqrt{5}}$ $\cos c = \frac{2}{\sqrt{5}}$, $\sin d = \frac{1}{\sqrt{10}}$ $\cos d = \frac{3}{\sqrt{10}}$ (a) sin (c + d) = sinc cos$d$ + cosc sin$d$ $= \frac{1}{\sqrt{5}} \times \frac{3}{\sqrt{10}} + \frac{2}{\sqrt{5}} \times \frac{1}{\sqrt{10}} = \frac{5}{\sqrt{50}} = \frac{1}{\sqrt{2}}$ (b) sin 2c = 2sinc cosc $= 2 \times \frac{1}{\sqrt{5}} \times \frac{2}{\sqrt{5}} = \frac{4}{5}$ cos 2d = 2cos^2d - 1 $= 2 \times \left(\frac{3}{\sqrt{10}}\right)^2 - 1 = \frac{8}{10} = \frac{4}{5}$ Example 2.16 on page 36 Questions 9 and 10 on page 37
7.	A curve has the equation $y = 3\sin x + \cos x$ (a) $k\sin(x-a) = k\sin x\cos a - k\cos x\sin a$ 3sin$x$ + cos x -$k$sin$a$ = 1, $k = \sqrt{10}$, $k$cos$a$ = 3 $\tan a = \frac{-1}{3}$, $a = 341.6^\circ$ y = $\sqrt{10}$ sin (x – 341.6)° (b) The maximum value of $y = 3\sin x + \cos x$ is $\sqrt{10}$ when x = 90° + 341.6° = 431.6° = 71.6° Example 2.32 on page 50 Questions 3 and 4 on page 52 Example 3.25 on page 76 Question 7 on page 77
8.	(a) $\cos\left(x - \frac{\pi}{6}\right) = \cos x \cos\frac{\pi}{6} + \sin x \sin\frac{\pi}{6} = \frac{\sqrt{3}}{2}\cos x + \frac{1}{2}\sin x$ (b) $\cos\frac{\pi}{12} = \cos\left(\frac{\pi}{4} - \frac{\pi}{6}\right) = \cos\frac{\pi}{4}\cos\frac{\pi}{6} + \sin\frac{\pi}{4}\sin\frac{\pi}{6} = \frac{1}{\sqrt{2}} \times \frac{\sqrt{3}}{2} + \frac{1}{\sqrt{2}} \times \frac{1}{2} = \frac{\sqrt{3}+1}{2\sqrt{2}}$ Example 2.11 on page 34 Question1e and f on page 36

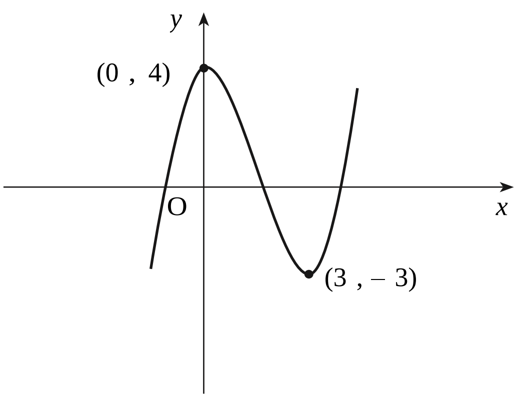
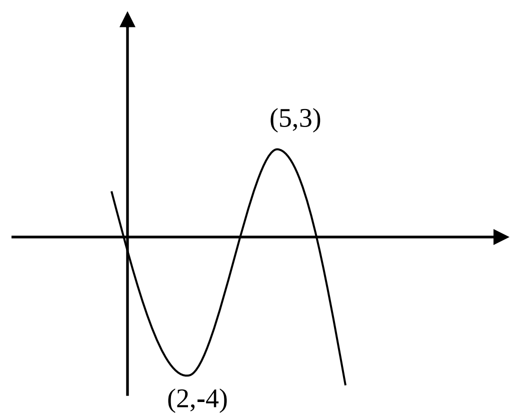
<table>
<tr><td>9.</td><td>$f(x) = \sqrt{9-2x}$ (a) $9 - 2x > 0, \quad x < 9/2$ (b) $f^{-1}(x) = \dfrac{9-x^2}{2}$

Example 4.2 on page 84   Example 4.6 on page 89   Question 1 on page 89</td></tr>
<tr><td>10.</td><td>Two functions are given as $f(x) = x^2 - 1$ and $g(x) = 2 - x$

(a) $f(g(x) = f(2-x)) = (2-x)^2 - 1$
$= 4 - 4x + x^2 - 1$
$= x^2 - 4x + 3,$

(b) $x^2 - 4x + 3 = 8, \quad x^2 - 4x - 5 = 0, \quad (x-5)(x+1) = 0, \quad x = 5$ or $x = -1$

Example 4.3 on page 86   Question 6 and 7 on page 88</td></tr>
<tr><td>11.</td><td>$a = 2, \; b = 3, \; c = -1$ $y = 2\sin(3x) - 1$

Example 3.24 on page 75   Questions 4 and 5 on page 77</td></tr>
<tr><td>12.</td><td>TP (0,4) $\rightarrow$ (2,-4)   TP (3,-3) $\rightarrow$ (5, 3)

Example 3.2 on page 56   Question 3 on Page 58</td></tr>
<tr><td>13.</td><td>The x-intercept (1,0) has moved 3 places to the right so $b = 3$
Given that $y = \log_a(x-3)$ passes through the point (5,1)
$1 = \log_a(5-3)$
$1 = \log_a(2)$ so $a = 2$ $y = \log_2(x-3)$

Example 3.19 on page 71   Question 2 on page 73</td></tr>
<tr><td>14.</td><td>(a) $3(x+2)^2 + 1$
(b)   The range of this fucntion is $y \geq 1$

Example 3.4 on page 59   Question 2 on page 61
Example 4.1 on page 84   Question 1 on page 85</td></tr>
</table>

15.	(a) $\boldsymbol{u} = \begin{pmatrix} -3 \\ 0 \\ 4 \end{pmatrix}$, $\lvert\boldsymbol{u}\rvert = \sqrt{(-3)^2 + 0^2 + 4^2} = 5$ (b) unit vector parallel to $\boldsymbol{u}$ is $\frac{1}{5}\begin{pmatrix} -3 \\ 0 \\ 4 \end{pmatrix}$ *or* $\begin{pmatrix} -3/5 \\ 0 \\ 4/5 \end{pmatrix}$ Example 5.3 on page 96 Question 10 on page 98
16.	$\boldsymbol{p} = \boldsymbol{a} + \frac{4}{5}\overrightarrow{AN} = \begin{pmatrix} -5 \\ 6 \\ -5 \end{pmatrix} + \frac{4}{5}\begin{pmatrix} 15 \\ -10 \\ 5 \end{pmatrix} = \begin{pmatrix} 7 \\ -2 \\ -1 \end{pmatrix}$, P is the point (7, -2, -1) Example 5.10 on page 103 Questions 1 and 2 on page 104
17.	(a) G (0,2,2) (b) $\boldsymbol{p} = \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}$ and $\boldsymbol{q}\begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}$ (c) cos POQ = $\frac{p \bullet q}{\lvert p\rvert\lvert q\rvert} = \frac{0\times1+1\times2+1\times1}{\sqrt{2}\times\sqrt{6}} = \frac{3}{\sqrt{12}} = \frac{\sqrt{3}}{2}$, angle POQ is 30° Example 6.5 and 6.6 on page 121 Questions 4 on page 123
18.	Given that vectors $\boldsymbol{a}$, $\boldsymbol{b}$ and $\boldsymbol{c}$ are defined as follows $\boldsymbol{a} = \begin{pmatrix} 2 \\ 0 \\ -1 \end{pmatrix}$, $b = \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}$ and $\boldsymbol{c} = \begin{pmatrix} 0 \\ -1 \\ 1 \end{pmatrix}$ $\boldsymbol{a}\bullet\boldsymbol{b} = 2\times1+0\times2+-1\times1 = 1$ $\boldsymbol{a}\bullet\boldsymbol{c} = 1\times0+2\times-1+1\times1 = -1$ $\boldsymbol{a}\bullet\boldsymbol{b} + \boldsymbol{a}\bullet\boldsymbol{c} = 0$ since $\boldsymbol{a}\bullet\boldsymbol{b} + \boldsymbol{a}\bullet\boldsymbol{c} = \boldsymbol{a}\bullet(\boldsymbol{b}+\boldsymbol{c})$, hence $\boldsymbol{a}\bullet(\boldsymbol{b}+\boldsymbol{c}) = 0$ and so $\boldsymbol{a}$ is perpendicular to $\boldsymbol{b}+\boldsymbol{c}$ Example 6.4 on page 119 and Example 6.9 on page 126 Question 7 on page 120, Question 1 in page 127